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On The Number of Independent Channels in a Diversity System

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***Abstract*—In a receive diversity system the use of multiple antennas at one end of the link produces multiple channels. A useful, although ill-defined, metric for such a link is the number of independent channels provided. In this letter we discuss several candidate metrics and compare their utility. We show that most of the metrics available in the literature have limitations and can exhibit non-physical behaviour. In order to improve on their performance, we develop two novel measures for the number of independent channels based on the statistical construction of the channel and channel capacity.**

I. Introduction

Consider the classic situation of receiver diversity where a single transmit antenna is communicating with an M-element antenna array at the receiver. This is often described as a single-input multiple-output system (SIMO). For such a system, performance measures such as capacity, bit-error-rate (BER) or signal-to-noise ratio (SNR) depend on the nature of the M channels from transmitter (TX) to receiver (RX). It is well-known that correlation between the channels usually reduces performance. The effects of correlation can be largely characterised by the $M \times M$ correlation matrix, but a more pragmatic approach would be to develop a single measure of the *overall correlation*. Instead of measuring this overall correlation, we could approach the problem from the opposite viewpoint and ask, how many independent channels are there? The concept of the number of independent channels (NIC) has obvious importance both as a practical measure and as a source of theoretical insight into channel behaviour. Hence, in this paper we consider this fundamental question. Related work in this area includes [1]–[7].

To motivate our approach, consider two widely spaced receive antennas in a rich scattering environment. In this scenario we are comfortable in accepting that a single transmitter has two independent channels in which to communicate. If the two antennas are co-located then there is only one channel. Hence for small but non-zero separations we should expect to encounter a value of NIC such that $1 < \text{NIC} < 2$. Several metrics have been proposed to measure NIC [1]–[7] or similar concepts in the context of a SIMO system. Several previously defined metrics are ad-hoc in nature [1]–[4], some are inherently insensitive to the correlations between antennas [5], [6] and some have unreasonable physical properties. Hence, this apparently simple situation is largely unresolved and in this paper we develop two novel and improved NIC metrics.

To begin, we should enumerate some desirable properties of a NIC metric. These are

1) $1 \leq \text{NIC} \leq M$.
2) The metric should be simple to compute.
3) The metric should measure a physical quantity or have physical meaning.
4) The metric should have simple properties.
5) The metric should make physical sense in a range of channel conditions.

Since the number of independent channels is not a single, universally understood quantity there will be debate over which metric is best. For example in this paper we present two new metrics. One is purely a function of the channel and the second is capacity based. Since capacity is inherently dependent on SNR it can be argued that capacity cannot be used to measure the number of independent channels since this is independent of the noise at the receiver. Nevertheless, capacity is a well-known concept and its use is somewhat intuitive in this context. Another interesting issue is whether NIC should increase as more antennas are deployed. Clearly, if an antenna array is fixed and an additional antenna is added then NIC should increase. However, if an extra antenna is added and the antennas are rearranged, then there is no guarantee that NIC will increase. This is made clear from a simple example. Consider an equally spaced array of 3 antennas, labeled, from left to right, 1, 2 and 3. Now assume a fourth antenna is added but the 4 antennas are rearranged so that antennas 1,2 and 3,4 are co-located in the positions of the original antennas 1 and 3. In this scenario NIC must drop. It is clear that the behaviour of NIC is somewhat subtle and this perhaps explains why no simple measure is widely accepted at present.

The rest of the paper is laid out as below. Section II details the SIMO system and channel model. Section III summarizes

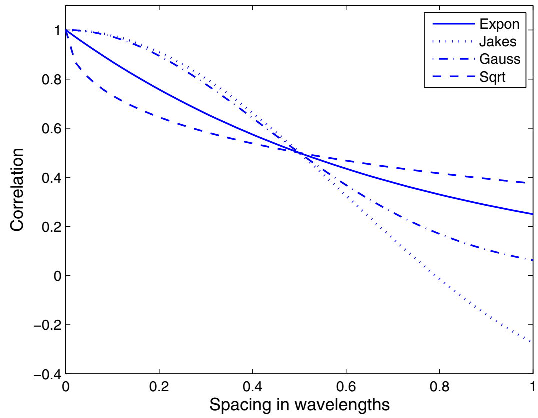

Fig. 1. The four correlation models parameterized by a decorrelation distance of 0.5λ.

previous NIC metrics and Sec. IV develops two new metrics. In Sec. V results are given and conclusions appear in Sec. VI.

II. SIMO System

For concreteness consider a SIMO system where the $M \times 1$ channel vector is denoted $\mathbf{h}$ and is complex Gaussian with zero mean vector and covariance matrix, $\mathbf{R}$. Notationally, we describe $\mathbf{h}$ as $\mathcal{CN}(\mathbf{0}, \mathbf{R})$. Hence, we are considering correlated Rayleigh fading. In most cases this assumption can be generalized but since it is the most important special case, a thorough investigation is warranted.

Assuming the array is not so widely spaced as to encounter different path loss or shadowing, we can assume without loss of generality that the diagonal elements of $\mathbf{R}$ are unity. The elements of $\mathbf{R}$ are denoted by $(\mathbf{R})_{ij} = r_{ij}$. There are a very large number of models for the correlation structure in a SIMO system. Here we consider four types.

1) Exponential correlation: $r_{ij} = a^{-d_{ij}}$, where d_{ij} is the distance between antennas i and j and $0 < a < 1$ [8].
2) The Jakes model: $r_{ij} = J_0(2\pi a d_{ij})$ [9].
3) A Gaussian decay model: $r_{ij} = a^{-d_{ij}^2}$.
4) A square root model: $r_{ij} = a^{-\sqrt{d_{ij}}}$.

In each case the model is parameterized by a single parameter, a, and the four models cover a range of scenarios from a continuous decay in the exponential to oscillation in the Bessel function. Sample curves are shown in Fig. 1, where Expon, Jakes, Gauss and Sqrt represent correlation types 1) - 4) respectively. Note that the correlation models are chosen for simplicity and coverage of a range of behaviours rather than physical reality. The correlation models shown in Fig. 1 have all been parameterized by a, so that the correlation drops to 0.5 at a spacing of half a wavelength. The spacing at which the correlation is 0.5 is denoted the "decorrelation distance" and can be used to set the value of a. The decorrelation distance is an arbitrary parameter. Simulation results in Sec. V use the value of 0.5. It is convenient to write the correlated channel vector, $\mathbf{h}$, in terms of an independent and identically distributed (iid) $\mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ vector $\mathbf{u}$, where $\mathbf{I}_M$ is the $M \times M$ identity matrix. Hence we have

$$\mathbf{h} = \mathbf{A}\mathbf{u} \tag{1}$$

where $\mathbf{A}$ is a lower triangular $M \times M$ matrix obtained from the Cholesky decomposition $\mathbf{R} = \mathbf{A}^\dagger \mathbf{A}$, where † denotes the Hermitian transpose. Two special cases are of particular interest. If the array is co-located then $\mathbf{R}$ is a matrix of ones, $\mathbf{R} = \mathbf{1}_M$, and NIC should be unity. For a widely separated array, $\mathbf{R} = \mathbf{I}_M$ and NIC should be M.

III. NIC Metrics

In this section we describe several metrics which have been proposed for NIC.

A. Ad-hoc metrics

In [4] a metric was suggested to capture correlation structure and measure the "power balance and degrees of freedom". The metric is defined by

$$\text{NIC}_1 = \frac{\text{tr}(\mathbf{R})}{\lambda_{max}(\mathbf{R})} \tag{2}$$

where $\text{tr}(\cdot)$ denotes trace and $\lambda_{max}(\cdot)$ is the maximum eigenvalue. In a similar vein an "effective degrees of freedom" metric was used in [2], defined by

$$\text{NIC}_2 = \frac{[\text{tr}(\mathbf{R})]^2}{\text{tr}(\mathbf{R}^2)} \tag{3}$$

Variations on NIC_1 and NIC_2 can be envisaged using different matrix norms as below

$$\text{NIC}_3 = \frac{\text{tr}(\mathbf{R})}{\|\mathbf{R}\|_\infty} \tag{4}$$

$$\text{NIC}_4 = \frac{[\text{tr}(\mathbf{R})]^2}{\|\mathbf{R}\|_F^2} \tag{5}$$

where $\|\mathbf{R}\|_\infty = \max_i\{\sum_{j=1}^M |r_{ij}|\}$ and $\|\mathbf{R}\|_F = \sqrt{\sum_{i=1}^M \sum_{j=1}^M |r_{ij}|^2}$ represent the infinity and Frobenius norm respectively. All four metrics satisfy the desired property that NIC $= 1$ for a co-located array and NIC $= M$ for a widely separated array.

B. Capacity based metrics

In a study of MIMO interference channels [3], the number of spatial degrees of freedom is defined as

$$\text{DOF} = \lim_{\rho \to \infty} \left\{ \frac{C(\rho)}{\log \rho} \right\} \tag{6}$$

where $C(\rho)$ is the capacity at SNR $= \rho$. This definition can also be used in the SIMO context. In [5] a general study of DOF and diversity (DIV) for SIMO and MIMO systems is undertaken. The resulting definitions are also in terms of high SNR and in this limiting regime, $\rho \to \infty$, neither DOF or DIV are related to $\mathbf{R}$. Hence, the DOF and DIV values are independent of correlation. Clearly this is not what is required here, where it is exactly the effects of correlation on the channels that is of interest. Hence we propose the non-limiting versions of DOF in [3] and [5] as

$$\text{NIC}_5 = \frac{\mathbb{E}[C(\rho, M)]}{\log \rho} \quad (7)$$

$$\text{NIC}_6 = \frac{\mathbb{E}[C(\rho, M)]}{\mathbb{E}[C(\rho, 1)]} \quad (8)$$

where $C(\rho, n)$ is the SIMO capacity with n antennas and SNR $= \rho$.

The values of the mean capacity for a spatially correlated SIMO system are well known [10] and can be written

$$\mathbb{E}[C(\rho, M)] = \frac{1}{\log 2} \sum_{j=1}^{M} b_j \exp\left(\frac{1}{\rho\lambda_j}\right) E_1\left(\frac{1}{\rho\lambda_j}\right) \quad (9)$$

where $\lambda_1, \lambda_2, \ldots, \lambda_M$ are the eigenvalues of $\mathbf{R}$, $E_1(\cdot)$ is the exponential integral, $E_1(x) = \int_1^\infty e^{-xt}/t \, dt$, and

$$b_j = \lambda_j^{M-1} \prod_{r \neq j} (\lambda_j - \lambda_r)^{-1}. \quad (10)$$

Note that the result in (9) is only valid when the matrix $\mathbf{R}$ has unequal eigenvalues. In the two special cases of interest where $\mathbf{R} = \mathbf{I}_M$ or $\mathbf{R} = \mathbf{1}_M$ we have the respective special cases [10]

$$\mathbb{E}[C(\rho, M)] = \frac{1}{M} \sum_{k=0}^{M-1} \frac{\exp(1/\rho)}{k!\rho^k} \left\{ (-1)^k E_1\left(\frac{1}{\rho}\right) + \sum_{r=1}^{k} (-1)^{k-r} \binom{k}{r} \rho^r \Gamma\left(r, \frac{1}{\rho}\right) \right\}, \quad (11)$$

$$\mathbb{E}[C(\rho, M)] = \exp(1/M\rho) E_1\left(\frac{1}{M\rho}\right), \quad (12)$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function. The result for $\mathbf{R} = \mathbf{1}_M$ follows from [10] since in this case the SIMO capacity expression collapses to a SISO expression which is a special case of [10].

Note from (11) and (12) that NIC does not vary between 1 and M as in the first four metrics. Also, by definition it varies with the SNR and is not therefore solely a function of the channel.

C. Receiver based metrics

In [7] it was proposed to think of diversity as equivalent to the shape parameter of the combiner output in a maximum ratio combiner (MRC). In MRC, the output SNR is given by $\gamma = \mathbf{h}^\dagger \mathbf{h} = \mathbf{u}^\dagger \mathbf{A}^\dagger \mathbf{A} \mathbf{u} = \mathbf{u}^\dagger \mathbf{R} \mathbf{u}$, using (1). Using the eigenvalue decomposition of $\mathbf{R}$ we can also write $\gamma = \tilde{\mathbf{u}}^\dagger \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M) \tilde{\mathbf{u}}$ where $\tilde{\mathbf{u}}$ is another iid, $M \times 1$, $\mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ vector and diag$(\cdot)$ represents a diagonal matrix. Finally, we have the result

$$\gamma = \sum_{i=1}^{M} \lambda_i |\tilde{\mathbf{u}}_i|^2. \quad (13)$$

In an iid fading channel $\lambda_1 = \lambda_2 = \cdots = \lambda_M = 1$ and γ has a Chi-squared distribution with shape parameter M. In [7] it is suggested that in the correlated fading case also, a gamma approximation to γ will have a corresponding shape parameter which can be interpreted as a measure of diversity. Applying this idea here, we note that

$$\mathbb{E}[\gamma] = \sum_{i=1}^{M} \lambda_i, \;\; \text{Var}[\gamma] = \sum_{i=1}^{M} \lambda_i^2. \quad (14)$$

The standard method of moments [11] approach to fitting the gamma shape parameter is to set the shape parameter equal to the ratio of $\mathbb{E}[\gamma]^2$ and Var(γ). Hence, we have a measure of diversity, or the number of independent channels, given by

$$\begin{aligned} \text{NIC} &= \frac{\mathbb{E}[\gamma]^2}{\text{Var}[\gamma]} &(15)\\ &= \frac{[\sum_{i=1}^{M} \lambda_i]^2}{\sum_{i=1}^{M} \lambda_i^2} &(16)\\ &= \frac{\text{tr}(\mathbf{R})^2}{\text{tr}(\mathbf{R}^2)}. &(17) \end{aligned}$$

Note that this is the same as NIC$_2$. Hence the ad-hoc metric has justification from this viewpoint.

The second receiver based metric we propose is a decorrelating receiver which processes the inputs to yield uncorrelated outputs, but has no power scaling. Hence, the channel vector $\mathbf{h} = \mathbf{A}\mathbf{u}$ is linearly processed by the weight matrix $\mathbf{W} = \frac{\sqrt{M}\mathbf{A}^{-1}}{\|\mathbf{A}^{-1}\|}$ which satisfies $\|\mathbf{W}\|^2 = M = \|\mathbf{I}_M\|^2$. The output is

$$\mathbf{W}\mathbf{h} = \frac{\sqrt{M}}{\|\mathbf{A}^{-1}\|} \mathbf{u} \quad (18)$$

which is an independent channel vector. The total power of $\mathbf{W}\mathbf{h}$ can be used as a new metric defined by

$$\text{NIC}_7 = \frac{M\mathbb{E}[\mathbf{u}^\dagger \mathbf{u}]}{\|\mathbf{A}^{-1}\|^2} = \frac{M^2}{\sum_{i=1}^{M} \frac{1}{\lambda_i^2}}. \quad (19)$$

Although intuitively a nice idea this metric has problems since one very small eigenvalue leads to NIC ≈ 0. This behaviour is similar to that of other metrics, such as entropy, which depend on det$(\mathbf{R})$ which vanish when an eigenvalue is zero. Hence, this metric varies between 0 and M. A rescaling could be performed to force the metric to lie between 1 and M but this would be rather artificial. Hence this metric is not considered further.

IV. Proposed Metrics

A. Construction metric

Note that the vector $\mathbf{h}$ is constructed via the equation $\mathbf{h} = \mathbf{A}\mathbf{u}$ where $\mathbf{A}$ is lower triangular. The iid vector $\mathbf{u}$ can be thought of as the input consisting of M independent channels. The first two elements of $\mathbf{h}$ can be written as

$$\begin{aligned} h_1 &= A_{11} u_1 &(20)\\ h_2 &= A_{21} u_1 + A_{22} u_2. &(21) \end{aligned}$$

Hence, h_1 contains an independent channel of power $|A_{11}|^2$. The second element, h_2, contains part of the first channel, the

$A_{21}u_1$ term, and an independent component of power $|A_{22}|^2$. Similarly we have

$$h_r = \sum_{i=1}^{r-1} A_{ri}u_i + A_{rr}u_r \tag{22}$$

and the r_{th} element has an independent component of power $|A_{rr}|^2$. In total, the vector $\mathbf{h}$ contains independent components of total power $\sum_{r=1}^{M} |A_{rr}|^2$. This is the proposed novel NIC metric

$$\text{NIC}_8 = \sum_{r=1}^{M} |A_{rr}|^2. \tag{23}$$

This metric has a physical interpretation as the total power of the independent components contained in $\mathbf{h}$. Since the metric is solely a function of the channel it has the advantage of being independent of SNR. The drawback of this approach is that it only applies to Gaussian channels where the scattered components have the form given in (1).

An interesting feature of the metric is that the Cholesky decomposition of $\mathbf{R}$ depends on the ordering of $\mathbf{h}$. Hence, different orderings of $\mathbf{h}$ give different values of the metric. At first sight, this non-uniqueness appears to be a problem. However, for physical reasons only one ordering (or set of equivalent orderings) is reasonable and the non-uniqueness problem disappears. To understand this property consider the fact that (20) - (22) themselves imply an ordering. Equations (20) and (21), for example, construct channel 2 after channel 1. Hence, whichever antenna is deemed to correspond to h_1 is considered first and the antenna corresponding to h_2 is considered second. For the metric to be physically reasonable it must satisfy certain properties. For example, NIC must increase if an extra antenna is added without any rearrangement of the existing antennas. Consider an array where antennas are to be placed at positions 1, 2 and 3. Positions 1 and 3 locate the ends of the array and position 2 is in the centre. Let NIC(i, j, k) denote the NIC value with antennas at positions i, j and k. Here, we must have NIC$(1, 2, 3) >$ NIC$(1, 3)$. The way to satisfy this constraint is to use the ordering 1,3,2 or 3,1,2. By symmetry these are equivalent. Also, since positions 1 and 3 are occupied first, the addition of location 2, using (22), adds an extra contribution to the existing NIC value. Hence, instead of 3! possible orderings giving different values of NIC, we have 2 possible equivalent orderings giving a unique answer. Similarly with 4 antennas and 5 antennas we have the possible orderings 1,4,2,3 and 1,5,3,2,4 respectively using a similar labeling of antennas. In fact, this natural construction ordering is also the ordering that maximizes NIC. This way of viewing the ordering also ensures that NIC increases when extra antennas are added without rearrangement.

As for metrics 1-4, the new metric ranges from 1 to M. A simple lower bound exists for the construction metric in the common case of the exponential correlation model. In this scenario, $r_{ij} = a^{-d_{ij}}$ and in the case of a uniformly spaced array, $r_{ij} = a^{-d|i-j|}$, where $d = d_{12}$. For this correlation matrix it can be shown that the Cholesky decomposition has leading diagonal given by $(1, \sqrt{1-\alpha^2}, \sqrt{1-\alpha^2}, \ldots, \sqrt{1-\alpha^2})$, where $\alpha = a^{-d}$. Proof is straightforward by induction. Hence we have the lower bound

$$\text{NIC}_8 \geq 1 + (M-1)(1-\alpha^2) = 1 + (M-1)\alpha^2. \tag{24}$$

This bound satisfies NIC$_8 = 1$ for $\alpha = 1$ and NIC$_8 = M$ for $\alpha = 0$. However, it is a lower bound since the original ordering of the antennas is used rather than the optimal, construction ordering.

B. Capacity metric

We now propose a second metric which is based on the capacity of the correlated system under consideration. Let the capacity of an M antenna SIMO system with correlation $\mathbf{R}$ and SNR ρ be $C(\rho, M)$. We define the number of independent channels, NIC$_9$, by the number of receive antennas in an uncorrelated system of equal ergodic capacity and equal SNR ρ. Since $C(\rho, M)$ will not necessarily correspond to an iid SIMO system with an integer number of antennas, we have

$$\mathbb{E}[C_{\text{iid}}(\rho, m)] \leq \mathbb{E}[C(\rho, M)] \leq \mathbb{E}[C_{\text{iid}}(\rho, m+1)] \tag{25}$$

where we have defined the capacity of an m antenna iid SIMO system as $C_{\text{iid}}(\rho, m)$. To determine an equivalent number of antennas, we linearly interpolate between the two iid systems, which has the physical interpretation of switching between m and $m+1$ antennas.

A drawback of any capacity based metric is that capacity increases with extra antennas even if the channel gains are identical. Hence capacity is measuring array gain as well as the impact of independent channels. In order to account only for the contribution of independent channels to the system, we apply a correction factor to the capacities to remove the contribution of the array gain. The corrected iid and correlated capacities are given by

$$\begin{aligned} \mu_{\text{iid}}(\rho, m) &= \mathbb{E}[C_{\text{iid}}(\rho, m)] - \Delta_m \\ \mu(\rho, M) &= \mathbb{E}[C(\rho, M)] - \Delta_M \end{aligned} \tag{26}$$

where the correction factor Δ_j for j antennas and operating SNR ρ can be shown from (12) to be

$$\Delta_j = \exp\left(\frac{1}{\rho j}\right) E_1\left(\frac{1}{\rho j}\right) - \exp\left(\frac{1}{\rho}\right) E_1\left(\frac{1}{\rho}\right). \tag{27}$$

The factor Δ_j can be identified as the growth in capacity over a single antenna system solely due to adding extra, identical channels. With this notation we define the metric NIC$_9$ as

$$\text{NIC}_9 = m + \frac{\mu(\rho, M) - \mu_{\text{iid}}(\rho, m)}{\mu_{\text{iid}}(\rho, m+1) - \mu_{\text{iid}}(\rho, m)}. \tag{28}$$

The version of (28) which includes the array gain contribution can be obtained from (26) and (28) with $\Delta_j = 0$.

V. Discussion and Results

Under normal conditions, say an array with half wavelength spacings, the different metrics can all behave reasonably and it is difficult to argue their relative merits. Hence, we consider the realistic case where increasing numbers of antennas may be employed but the array length is fixed. In particular, we consider a linear array with a length of one wavelength and equal spacing between the antennas. The decorrelation distance is set to a half wavelength. For large numbers of antennas the array becomes densely packed and this provides a more rigorous test of whether the metrics provide physically plausible results. This situation is interesting in its own right and there has been work on the capacity of such arrays [12]–[14]. In this work we ignore coupling effects since the focus is on statistical channel models.

Results for this scenario are shown in Figs. 2 to 7. Figure 2 shows that all four ad-hoc metrics reach a peak value for NIC and then decay away in the case where the correlation is exponential. Although not shown, this decay is even more noticeable for the Gaussian and Jakes correlation models. It is difficult to argue that NIC decreases steadily after a few antennas are deployed. Hence these metrics are not recommended.

Figure 3 shows the sensitivity of the capacity based metrics to SNR. Note that for NIC_5 the number of independent channels varies from 1.8 to 4.8 as the SNR moves from 0 dB to 20 dB with 20 antennas. This is a large sensitivity to SNR which is an undesirable feature in a NIC metric. Furthermore, the precise meaning of such metrics is not clear.

The new construction metric is shown in Figs. 4 and 5. For the correlation models which decay rapidly close to the origin (the square root and exponential decay models) the NIC metric increases steadily and begins to plateau. For the correlation models which are smooth at the origin (the Gaussian and Jakes models) the behaviour is different. For odd numbers of antennas and for even numbers of antennas, the same rise and leveling off is observed. However, as you move from an even number to an odd number of antennas the NIC can drop. This is reasonable since extra antennas will not always increase NIC, it will depend on their placement. When you move from an even number to an odd number of antennas, the actual placement of all the antennas changes, with the exception of the outer antennas. Hence, this behaviour is plausible. Furthermore, these results have considerable implications for the use of statistical channel models in closely packed arrays. The behaviour of the correlations at small separations is critical and the use of different models may lead to fundamentally different behaviour.

Finally, the new capacity based metric, NIC_9, is shown for exponential and Jakes correlation models in Figs. 6 and 7, respectively. The figures were obtained for a decorrelation distance of 0.5 and $\rho = 5$ and 20 dB. Included in the figures is NIC computed without the array gain correction factor, that is using with $\Delta_m = \Delta_M = 0$ in (26). The figures clearly demonstrate the need for the correction factor. Without the correction factor, NIC increases approximately linearly for the values of M considered. A more reasonable metric is obtained with the correction factor, where the NIC reaches a steady state value of $2-3$ for a system with $M \approx 6$ with the parameters considered. Although the correction factor removes the continual growth in NIC it also makes NIC more SNR dependent and in Fig.7 we observe a slight but steady decay in NIC with an SNR of 20 dB. Clearly there are still some

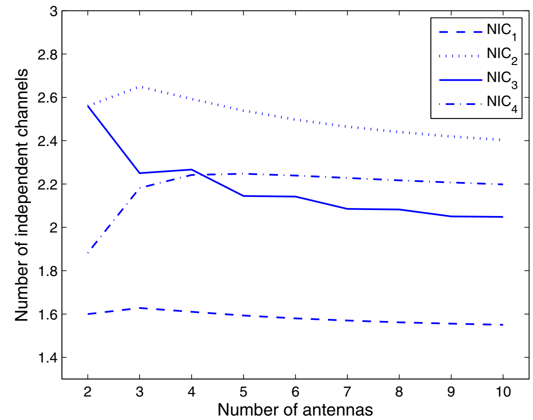

Fig. 2. NIC vs antenna numbers for metrics 1-4 with the exponential correlation model.

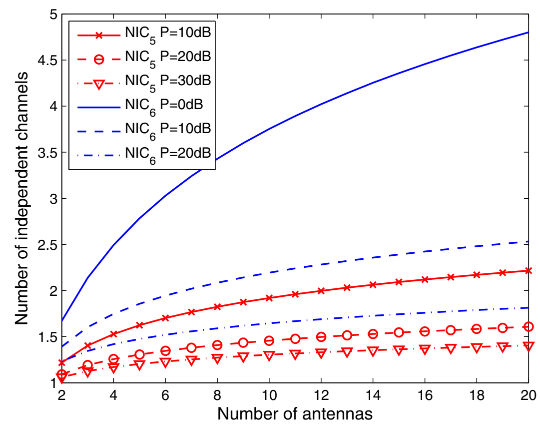

Fig. 3. NIC vs antenna numbers for metrics 5 and 6 with the exponential correlation model.

concerns over the use of capacity based NIC metrics.

VI. Conclusions

We considered the problem of quantifying the effective number of independent channels in a spatially correlated SIMO system. We discussed a number of previously proposed metrics and introduced two novel metrics to address their shortcomings.

We proposed a construction metric based on the Cholesky decomposition of the correlation matrix. The metric has a physical interpretation as the total power contained in the independent components of the channel. The metric returns a plausible NIC for a variety of correlation models considered, while highlighting the importance of antenna placement on the metric output.

The second metric introduced defines NIC as the number of receive antennas in an uncorrelated SIMO system of equal capacity. A correction factor was introduced to remove the effects of array gain from the capacity expression. For the exponential and Jakes correlation models the resulting metric

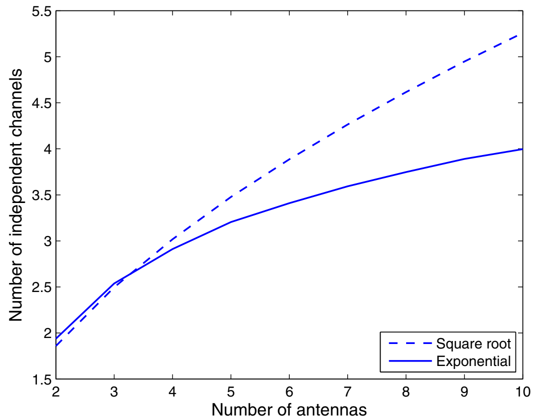

Fig. 4. NIC vs antenna numbers for metric 8 with various correlation models.

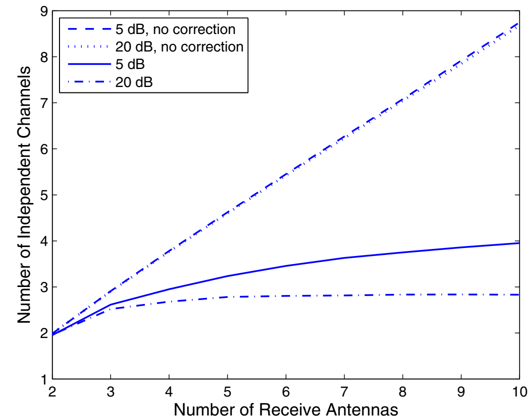

Fig. 6. NIC vs antenna numbers for metric 9 with the exponential correlation model. Corrected and uncorrected versions of the metric are shown.

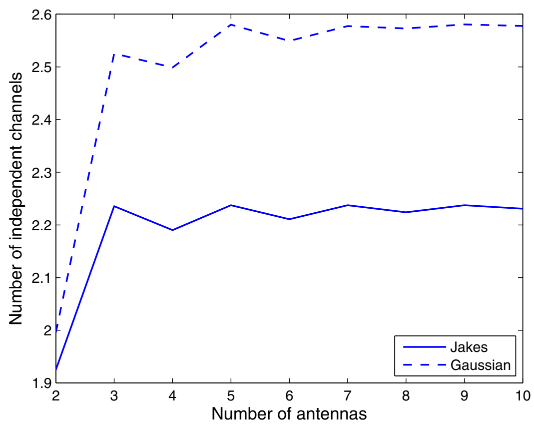

Fig. 5. NIC vs antenna numbers for metric 8 with various correlation models.

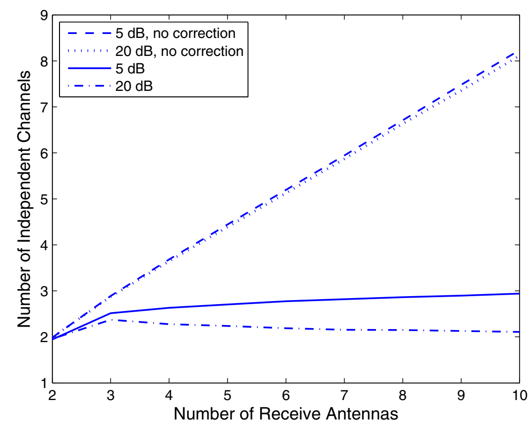

Fig. 7. NIC vs antenna numbers for metric 9 with the Jakes correlation model. Corrected and uncorrected versions of the metric are shown.

was shown to be well behaved, showing no significant fluctuation with SNR and a moderate increase with the introduction of additional antennas to a fixed size array. We note that both of the proposed NIC metrics can also be applied to MIMO channels. However, for reasons of space, these extensions are left to an extended journal version of this paper.

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